Observability and Observer Design for Hybrid Systems

Elena DE SANTIS

UNIVERSITY OF L’AQUILA (I)
Dep. of Electrical Eng. and Information Science, Center of Excellence DEWS
(Design of Embedded systems, Wireless interconnect and System on chip)

Work supported by HYCON Network of Excellence and by Ministero dell’Istruzione, dell’Università e della Ricerca, Projects MACSI and SCEF (PRIN05).

Observability for interconnected systems
February 13, 2008
Motivation:

Simplified Representation of Information Flow in ATC:

... local notion of observability needed!

Input-Output Interconnection of LSw-systema:

Given $S_1$ and $S_2$:

- Inputs of $S_1$ = Inputs of $S_2$
- Outputs of $S_1$ = Outputs of $S_2$
- Outputs of $S_1$ = Inputs of $S_2$

Input-Output Interconnection of $S_1$ and $S_2$ is a LSw-sys. $S = S_1 || S_2$ such that:
Input-Output Interconnection of LSw-sys:

Given two SLSs $S_1$ and $S_2$ consider $S = S_1 \parallel S_2$

External variables:
- Hybrid Input of $S$:
  - Discrete Disturbance $\sigma_1$
  - Continuous Control Input $u_1$
- Hybrid output of $S$

Internal variables:
- Hybrid State $(q_1, x_1)$ of $S_1$
- Hybrid State $(q_2, x_2)$ of $S_2$
- Latent Hybrid Variables $(p_1, y_1) = (p_2, y_2)$

Variables in Interconnected LWs-sys:

... generation of $\varepsilon$-transitions!
Observability of Internal Variables of $S_1||S_2$

Consider $S = S_1||S_2$

An internal variable $v$ is **observable** from $S_1||S_2$ if there exists $u_i$ of $S$ such that it is possible to reconstruct the evolution in time of $v$ from $(p_2,y_2)$ of $S$, for any $t \neq t_j$, $j=0,1,...$.

S$_1$ is observable

S$_2$ is observable

S$_1||S_2$ is not observable!
Observability Notions Hierarchy

Observability of $q_1$ and $q_2$: 
$q_1$ is observable from $S_1||S_2$ iff $(q_1, q_2), (q_1', q_2')$

$$q_1 \neq q_1' \Rightarrow T(S(q_1)||S(q_2))(s) \neq T(S(q_1')||S(q_2))(s)$$

Observability of $(q_1, x_1)$ and $(q_2, x_2)$: 
Assume that $S_1||S_2$ location observable. 
$(q_1, x_1)$ is observable from $S_1||S_2$ iff $(q_1, q_2) \in Q_1 \times Q_2$

$$\text{Ker}(O) \subseteq \{0\} \times R^{n(2)}$$

where $O$ is the observability matrix associated with $S(q_1)||S(q_2)$
**Observability of Internal Variables in S_1||S_2**

**Observability of discrete latent internal variable p_i:**

p_i is observable from S_1||S_2 if

- \(((q_1, q_2), \sigma, (q'_1, q'_2)) \in \gamma^{-1}(\epsilon) \subseteq E\)
  \(\Rightarrow T(S(q_1)||S(q_2))(s) \neq T(S(q'_1)||S(q'_2))(s)\)

- \(p_2 \in P_2,\ e_1, e_2 \in \gamma^{-1}(p_2) \subseteq E, \text{s.t.}\)
  \(e_1 = ((q_1, q_2), \sigma_1, (q_3, q_4)), e_2 = ((q_5, q_6), \sigma_1, (q_7, q_8)),\)
  \(e_1 \neq e_2 \text{ and } \gamma(q_1, \sigma, q_3) \neq \gamma(q_5, \sigma, q_7)\)
  one of the following conditions holds:

- \(T(S(q_1)||S(q_2))(s) \neq T(S(q_5)||S(q_6))(s)\)
- \(T(S(q_3)||S(q_4))(s) \neq T(S(q_7)||S(q_8))(s)\)

Reconstruction of the switching time

Reconstruction of the latent variable

**Observability of hybrid latent internal variable (p_i, y_i):**

Assume that S_1||S_2 is location observable.

(p_i, y_i) is observable from S_1||S_2 iff \((q_1, q_2) \in Q_1 \times Q_2: \)

\[\text{Ker}(O) \subseteq \text{Im}([C_1(q_1) 0]')\]

where:

- O observability matrix of S(q_1)||S(q_2)
- M' transpose of M