**Presence and Observer Design for Hybrid Systems**

**Elena DE SANTIS**

UNIVERSITY OF L'AQUILA (I)  
Dep. of Electrical Engineering, Center of Excellence DEWS  
Design of Embedded Systems, Wireless Interconnect and System on chip

Work supported by HYCON Network of Excellence and by Ministero dell'Istruzione, dell'Università e della Ricerca, Projects MACSI and S2EF (PRIN05).

**Observed Predicates**

If the observability matrix has full rank, then:

- the system is observable;
- a Luenberger observer, with desired error dynamics, can be designed.

**Stability and Observer design**

February 12, 2008

**Observers for Continuous LTI Systems**

\[
\dot{z}(t) = A \dot{x}(t) + B \dot{u}(t) \\
y(t) = C \dot{x}(t)
\]

\[
\dot{z}(t) = (A - GC) \dot{z}(t) + B \dot{u}(t) + G \dot{y}(t)
\]

\[
e(t) = \dot{z}(t) - \dot{x}(t) \\
\dot{e}(t) = (A - GC) e(t)
\]

If the observability matrix has full rank, then

- the system is observable;
- a Luenberger observer, with desired error dynamics, can be designed.

**Observers for Switching LTI Systems**

\[
\dot{z}(t) = A \dot{x}(t) + B \dot{u}(t) \\
y(t) = C \dot{x}(t)
\]

\[
\dot{z}(t) = (A - GC) \dot{z}(t) + B \dot{u}(t) + G \dot{y}(t)
\]

**Stability of Switching Systems**

\[
A = \begin{pmatrix} -0.1 & -4 \\ 1 & -0.1 \end{pmatrix}
\]

\[
A = \begin{pmatrix} -0.1 & -1 \\ 4 & -0.1 \end{pmatrix}
\]

\[
\dot{z}(t) = A \dot{x}(t) \\
x(t) = e^{At} \\
e^{At} = I + At + A^2 t^2/2 + A^3 t^3/6! + ...
\]
Globally exponentially stable if:

- A common Lyapunov function exists [Johansson, TAC’98]
- The Lie algebra \([F_i]\) is solvable [Narendra, TAC’94; Liberzon SCL’99]
- There is a dwell time, where \(t \geq 0\) [Morse, TAC’96]

\[
\dot{x}(t) = F_i x(t)
\]


Discrete-time LSW-systems, without reset and with known discrete state:

\[
\dot{x} = A_{c,i} x + B_{c,i} u + L_{c,i} (y - C_{c,i} x), \quad t \geq 0
\]

Th. Assume the pairs \((A_i, C)\) observable, \(i=1...N\). If there exists a symmetric positive definite matrix \(P\) as the solution of the algebraic Lyapunov inequalities

\[
(A_i - L_i C)'P(A_i - L_i C) - P < 0, \quad i = 1 ... N
\]

then the error asymptotically converges to zero.


Continuous-time LSW-systems, without reset and with unknown discrete and continuous state.

Continuous-time observer scheme.

Hybrid Observer Scheme

Discrete input \(\sigma(t)\) Continuous input \(u(t)\)
Plant Hybrid Model \(q(t), x(t)\) Continuous output \(\tilde{y}(t)\)
Discrete output \(\psi(t)\)
Continuous Observer
Location Observer
Hybrid Observer

\(\tilde{x}(t)\)
Hybrid Observer Scheme

- Discrete input $\sigma(A)$
- Continuous input $\sigma(\delta)$
- Plant Hybrid Model $\psi(q(k),x(k))$
- Location Observer
- Continuous output $\eta(k)
- Discrete output $\phi(k)$
- Hybrid Observer
- Location observability $\bar{z}(k)$

Hybrid Observer Scheme: extracting discrete dynamics!

- Discrete input $\sigma(A)$
- Continuous input $\sigma(\delta)$
- Plant Hybrid Model $\psi(q(k),x(k))$
- Location Observer
- Continuous output $\eta(k)
- Discrete output $\phi(k)$
- Hybrid Observer
- Location observability $\bar{z}(k)$

Hybrid Observer Scheme: extracting discrete dynamics!

- Discrete input $\sigma(A)$
- Continuous input $\sigma(\delta)$
- Plant Hybrid Model $\psi(q(k),x(k))$
- Location Observer
- Continuous output $\eta(k)
- Discrete output $\phi(k)$
- Hybrid Observer
- Location observability $\bar{z}(k)$

Hybrid Observer Scheme: extracting discrete dynamics!

- Discrete input $\sigma(A)$
- Continuous input $\sigma(\delta)$
- Plant Hybrid Model $\psi(q(k),x(k))$
- Location Observer
- Continuous output $\eta(k)
- Discrete output $\phi(k)$
- Hybrid Observer
- Location observability $\bar{z}(k)$

FSM Current-State Observability [Ramadge 1986]

An alive FSM

$q (k+1) \in \varphi (q(k), \sigma (k+1))$

$\sigma (k+1) \in \phi (q(k))$

$\psi (k+1) \in \eta (q(k), \sigma (k+1), q(k+1))$

is said to be current-state observable if there exists an integer $K$

such that for every $k > K$:

- for any unknown initial state $q(0)$ and
- for any input sequence $\sigma(1), \sigma(2)$

the state $q(i)$ can be determined

- from the observation sequence $\psi(i), \psi(i+1), \ldots$.

E. De Santis: Hybrid Systems and Observability
Observers for FSMs

Given an FSM, an observer is

$$\sigma(k)$$

$$\psi(k)$$

$$q(k)$$

$$\text{Obs}$$

$$\bar{q}(k)$$

Theorem: An alive FSM is current-state observable if and only if there exists a nonempty subset $$E_O$$ of singletons in the observer FSM such that

- $$E_O$$ is invariant
- all cycles are contained in $$E_O$$

Persistent states

Assume that a null output event $$\varepsilon$$ may be generated. A state is termed persistent if it may be visited after an arbitrarily long string of input events.

Theorem: An alive FSM is current-state observable if and only if, for every persistent state $$q$$ of the FSM,

- (c1) all exit transitions of $$q$$ are observable,
- (c2) there exists a singleton state $$\{q\}$$ in the observer and it is the only persistent state of the observer containing $$q$$.

State Observation for Hybrid Systems

The plant hybrid model is

$$q(k+1) \in \psi(q(k), \sigma(k+1))$$

$$\sigma(k+1) \in \phi(q(k), x(t_{k+1}), u(t_{k+1}))$$

$$\psi(k+1) \in \eta(q(k), \sigma(k+1), q(k+1))$$

$$\dot{x}(t) = A_{q(k)} x(t) + B_{q(k)} u(t) + P \omega(t)$$

$$y(t) = C_{q(k)} x(t)$$

$$x(t_{k+1}) = R_{q(k), \sigma(k)} x(t_k) + R_{q(k), \sigma(k+1)} \omega(t)$$

Hybrid Observer Scheme

The plant hybrid observer is

$$\dot{\bar{q}}(k) = A_{\bar{q}(k)} \bar{q}(k) + B_{\bar{q}(k)} \bar{u}(k) + P_{\bar{q}(k)} \bar{w}(k)$$

$$\bar{y}(k) = C_{\bar{q}(k)} \bar{q}(k)$$

$$\bar{x}(t_{k+1}) = R_{\bar{q}(k), \bar{\sigma}(k)} \bar{x}(t_k) + R_{\bar{q}(k), \bar{\sigma}(k+1)} \bar{\omega}(t)$$
Assumptions

• **Living** hybrid system
  - All executions are non-Zeno (for any \( t \), \( \text{card}\{ t_j : t_j < t \} < \infty \))
  - All executions have an infinite number of transitions
    (e.g. Nonblocking HS with min and max dwell time)
• All time bases: \( t' - t > 0 \), i.e. no multiple instantaneous transitions

Current-Location Observable Hybrid Systems

If the FSM

\[
q(k + 1) \in \varphi(q(k), \sigma(k + 1))
\]
\[
\sigma(k + 1) \in \bigcup_{c \in C} \phi(q(k), z, u)
\]
\[
\psi(k + 1) \in \eta(q(k), \sigma(k + 1), q(k + 1))
\]

associated to the hybrid plant is alive and current-state observable then the current location of the hybrid plant can be identified from the discrete output only.

Exponential convergence (for CLO)

A hybrid observer is said to be **exponentially ultimately bounded** if there exist a positive integer \( K \) and constants \( c \geq 1, \mu > 0 \) and \( b \geq 0 \), such that

\[
\| \hat{z}(t) - z(t) \| \leq \frac{\| x(t) \|}{c \mu} \quad \forall t \geq K
\]

for any hybrid initial state \( \{ q(0), x(0) \} \),

- any plant inputs \( \sigma(1), \sigma(k) \) and \( u(\tau), \tau = [0, t] \)
- any feasible observation sequence \( \psi(1), \psi(k) \).

\( \mu \) is the rate of convergence, \( b \) is the ultimate bound.

If \( b = 0 \) the observer is said to be **exponentially convergent**.

Location Observer: discrete observer of the plant FSM

Continuous Observer: switched Luenberger observer with resets and switching controlled by the identified plant location

\[
\dot{\hat{z}} = (A_2 - GC) \hat{z} + B_2 u + G_2 y
\]

\[
\hat{z} = 0, \quad \hat{z} = \psi(k)
\]

\[
\ddot{z} = 0, \quad \ddot{z} = \psi(k)
\]

\[
\hat{z} = (A_1 - G_1 C) \hat{z} + B_1 u + G_1 y
\]

\[
\hat{z} = (A_1 - G_1 C) \hat{z} + B_1 u + G_1 y
\]

\[
\hat{z} = (A_2 - G_2 C) \hat{z} + B_2 u + G_2 y
\]

\[
\hat{z} = (A_2 - G_2 C) \hat{z} + B_2 u + G_2 y
\]
General case: Not CLO Hybrid Systems

Hybrid system not CLO but current location observability can be achieved if:
- the difference between the CT dynamics in 2 and 4 can be identified
- some additional discrete outputs can be obtained by processing CT evolution (signatures)

Location Observer Design: general case

1/\(b\), \(s\) 4

0/\(b\), \(s\) 2

Transition detector

Transition detector has to recover current-location observability:
- by detecting the unobservable transition (if (C1) fails)
- by distinguishing transitions that yield to non-singleton persistent states in the observer that contain persistent states of \(H\) (if (C2) fails)
and producing a corresponding output event (signature)

Recovering Current-location Observability

- A living hybrid system \(H\) is said to be current-location recoverable if there exists a transition detector \(T\) such that the complementary FSM associated to the composition of \(H\) and \(T\) is current-state observable.
- \(T\) is an admissible transition detector if \(H\) composed with \(T\) is current-location observable.

Exponential ultimate boundedness (for not CLO)

Theorem: Consider current-location recoverable hybrid system with dwell time \(D > 0\). If:
- pairs \((A_i, C_i)\) associated with every persistent state \(q_i\) are observable,
- admissible transition detector identifies discrete location of \(H\) with delay less than \(\Delta\) with \(0 \leq \Delta < D\)
then, for any rate of convergence \(\mu > 0\), there exist gain matrices \(G_i\) so that hybrid observer exponentially ultimately bounded, with rate of convergence \(\mu > 0\).
**Signature generator**

- N’ Luenberger observers:

\[ z_i(t) = (A_i - L_i C_i) z_i(t) + B_i u(t) + L_i y(t) \]

\[ r^*_i(t) = C_i z_i(t) - y(t) \]

\[ s_i = \begin{cases} \text{true if } r^*_i(t) \leq \varepsilon \quad &\text{[Massoumnia, Verghese, Willsky, TAC 1989]} \\ \text{false if } r^*_i(t) > \varepsilon \end{cases} \]

- Estimator gains \( L_i \) can be chosen such that signatures are true before an arbitrary time \[ [\text{Balluchi et al., HSCC2000}] \]

**Actual Engaged Gear Identification**

[A. Balluchi, L. Benvenuti, C. Lemma, A. Sangiovanni - Vincentelli, G. Serra, IFAC2005]

- Motivation: improve drivability and tailpipe emission control
- A very detailed hybrid model of an automotive driveline
- Design of the hybrid observer for actual engaged gear identification
- Validation: extensive simulations and experimental data provided by Magneti Marelli Powertrain

**Exponential Ultimate Boundedness**

Convergence during the interval \( D - \Delta \) has to overcome divergence during the interval \( \Delta \) and overshoots

\[ x = T \]

Not smaller than \( D \)

Not greater than \( A \)

Desired exponential convergence

Error convergence band

**Hybrid Observer**

**Experimental Results (Opel-Astra)**

Second

First

Neutral

First gear residual

Second gear residual

Identified actual engaged gear by the hybrid observer