

Hybrid systems and control

Christophe Prieur and Andrew Teel

EECI Graduate Course 2008

Instructors: Christophe Prieur and Andrew Teel

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Course Home Page:

<http://www.laas.fr/~cprieur/Courses/eeci08.html>

It contains some useful informations, the content of the course, the schedule and some references. The references of the instructors may be downloaded (please ask for a password).

Course description:

Hybrid systems are systems modeled by discrete dynamics (e.g. a resetting subordinated to logic-even) combined with continuous dynamics (e.g. a physical parameter). Also, although many control systems are usually described by either a continuous-time model or a discrete-time model, when designing a stabilizing feedback, it may be necessary (or more efficient) to consider a hybrid feedback law (e.g with some flows and some jumps). In this context, the system in closed loop becomes hybrid.

In the first part of this course, we recall some notions and some tools for the stabilization of control systems by means of (dis)continuous state-feedbacks. In the second part, we introduce a class of hybrid systems and define a notion of solution for such systems. This leads to several engineering applications with the hybrid systems framework. We study the stability theories by focusing on the KLL stability, robust KLL stability, and necessary and sufficient conditions for stability of hybrid

systems. Several examples illustrate this part. In the third part of this course, we consider the design problem of hybrid feedbacks for nonlinear control systems. Some theoretic applications of these technics are given, including robust stabilization problems or optimal control theory. Also some applications are considered in robotics or in embedded systems. In the last part of this course, the problem of the output regulation by means of hybrid controllers is studied.

The course is suitable for engineering and mathematics students who are familiar with basic linear control system theory.

Program:

- Notions of stability of nonlinear control systems;
- Definition of hybrid systems, notion of solutions, stability with respect to perturbations;
- Design of hybrid feedbacks for nonlinear control systems. Theories and examples;
- Applications;
- Hybrid output feedback laws.

Outline of the course:

First part Classical system theory [CP - three sessions]

Second part Hybrid systems theory [AT - four sessions]

Third part Hybrid stabilizers [CP - three sessions]

Fourth part Further control applications [CP -one session and AT - three sessions]

Timetable:

The duration of the course is 21 hours. It is splitted into 14 sessions whith a duration of 1.5 hours each. In the following table, “AT” (resp. “CP”) means that the instructor is Andrew Teel (resp. Christophe Prieur) .

	Monday	Tuesday	Wednesday	Thursday	Friday
9:00am - 10:30am		Slot 3: CP	Slot 7: AT	Slot 9: CP	Slot 13: AT
11:00am - 12:30pm		Slot 4: AT	Slot 8: CP	Slot 10: CP	Slot 14: AT
2:00pm - 3:30pm	Slot 1: CP	Slot 5: AT		Slot 11: CP	
4:00pm - 5:30pm	Slot 2: CP	Slot 6: AT		Slot 12: AT	

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