Sliding Mode Observers for Automotive Alternator
Back EMF is to be found.

One of the methods for controlling the synchronous rectifier requires the detection of alternator's back electromotive forces (EMFs) by sensing the rotor position.

- packaging, cost and reliability issues
$R_w = 0.033 \Omega$
$L = 0.000155 H$
$C = 40 F$
$R_L = 0.12 \Omega$
$R_b = 0.012 \Omega$

System dynamics equations:

$$\frac{d}{dt} i_k = -\frac{R_w}{L} i_k - \frac{1}{6L} (2u_k - \sum_{j\neq k} u_j) v_o + \frac{1}{L} e_k$$

where

$$C \frac{R_L + R_b}{R_L} \frac{du_c}{dt} = -\frac{1}{R_L} u_c + \frac{1}{2} \sum_{k=1}^{3} i_k u_k$$

$$v_o = CR_b \frac{du_c}{dt} + u_c$$
Observers with Phase Current Measurements

\[ e_1(t) = A \sin(\omega t); \quad \omega \text{ is constant} \]

\[ \begin{cases} \dot{e}_1 = e_1' \\ \dot{e}_1' = -\omega^2 e_1 \end{cases} \]

- Observer 1 (sliding mode observer)

\[
\frac{d}{dt} i_1 = -\frac{R}{L} i_1 - \frac{v_0}{6L} (2u_1 - \sum_{j \neq 1} u_j) + \frac{1}{L} e_1' \\
\frac{d}{dt} \hat{i}_1 = -\frac{R}{L} \hat{i}_1 - \frac{v_0}{6L} (2\hat{u}_1 - \sum_{j \neq 1} \hat{u}_j) + \frac{1}{L} M_1 \text{sign}(i_1 - \hat{i}_1) \\
(M_1 \text{sign} (\hat{i}_1))_{eq} = e_1; \quad \hat{i}_1 = i_1 - \hat{i}_1 \\
\tau \dot{z} + z = M_1 \text{sign} (\hat{i}_1); \quad z \to e_1 \quad \text{when} \quad \tau \to 0
\]

- Observer 2 (linear asymptotic observer)

\[
\begin{cases} \dot{\hat{e}}_1 = \hat{e}_1' - L_{11}(\hat{e}_1 - z) \\ \dot{\hat{e}}_1' = -\hat{m} \hat{e}_1 - L_{21}(\hat{e}_1 - z) \end{cases}
\]

\[ \dot{\hat{m}} = L_{31} \bar{e}_1 \hat{e}_1; \quad \bar{e}_1 = \hat{e}_1 - z \]
Observers with Link Current Measurements

- **Observer 1** (sliding mode observer)

\[
\frac{d}{dt} \hat{i}_1 = -\frac{R_w}{L} \hat{i}_1 - \frac{v_o}{6L} (2u_1 - \sum_{j \neq 1} u_j) + \frac{1}{L} M_i \text{sign}(u_1i_{\text{link}} - \hat{i}_1)
\]

\[
(M_i \text{sign}(\hat{i}_1))_{eq} = e_i; \quad \bar{i}_1 = i_1 - \hat{i}_1
\]

\[
\tau \dot{z} + z = M_i \text{sign}(\bar{i}_1); \quad z \to e_1 \text{ when } \tau \to 0, \text{ within the window.}
\]

- **Observer 2** (linear asymptotic observer)

\[
\begin{align*}
\hat{e}_1 &= \hat{e}_1' - L_{11}(\hat{e}_1 - z) \\
\hat{e}_1' &= -\hat{\omega}^2 \hat{e}_1 - L_{21}(\hat{e}_1 - z) \\
\dot{\hat{\omega}} &= L_{31} \bar{e}_1 \hat{e}_1; \quad \bar{e}_1 = \hat{e}_1 - z
\end{align*}
\]

Note: Observer 2 works only within the window when \( u_1 \neq u_2 = u_3 \)

otherwise, \( L_{11} = L_{21} = L_{31} = 0 \)
Figure 9: The estimated back EMF obtained from the LAO taking into account that $z \rightarrow e_1$ for $\delta = 0.1 ms$, $L_{11} = 4000$, $L_{21} = 4,000,000$ and positive $L_{31} = 20,000$

Figure 10: The estimated frequency obtained from the LAO taking into account that $z \rightarrow e_1$ for $\delta = 0.1 ms$, $L_{11} = 4000$, $L_{21} = 4,000,000$ and positive $L_{31} = 20,000$
Nonlinear Asymptotic Observer

- Time-varying frequency (Variable speed) \( \omega(t) = \alpha t + \beta \)

\[
e_1(t) = A(\alpha t + \beta) \sin(\alpha \frac{t^2}{2} + \beta t + \gamma)
\]

EMF Model \[
\begin{align*}
\dot{e}_1 &= e_1' \\
\dot{e}_1' &= -(\omega^2 + \frac{3\alpha^2}{\omega^2}) e_1 + \frac{3\alpha}{\omega} e_1' \\
\dot{\omega} &= \alpha \\
\dot{\alpha} &= 0
\end{align*}
\]

- Nonlinear asymptotic observer (if \( \omega(t) \) is measured)

\[
\begin{align*}
\dot{\hat{e}}_1 &= \hat{e}_1' - L_{11}(\hat{e}_1 - z) \\
\dot{\hat{e}}_1' &= -(\hat{\omega}^2 + \frac{3\hat{\alpha}^2}{\hat{\omega}^2}) \hat{e}_1 + \frac{3\hat{\alpha}}{\hat{\omega}} \hat{e}_1' - L_{21}(\hat{e}_1 - z) \\
\dot{\hat{\omega}} &= \hat{\alpha} - k_{11}(\hat{\omega} - \omega) \\
\dot{\hat{\alpha}} &= -k_{21}(\hat{\omega} - \omega)
\end{align*}
\]

Note: The observer works only within the window when \( u_1 \neq u_2 = u_3 \)
otherwise, \( L_{11} = L_{21} = 0 \)
Nonlinear Asymptotic Observer

- NAO for the case of link current measurement
  (if $\omega(t)$ and $\alpha$ are both unknown, i.e. without a speed sensor)

\[
\begin{align*}
\dot{\hat{e}}_1 &= \dot{\hat{e}}_1 - L_{11}(\hat{e}_1 - z) \\
\dot{\hat{e}}'_1 &= -\left(\hat{\omega}^2 + \frac{3\hat{\alpha}^2}{\hat{\omega}^2}\right)\hat{e}_1 + \frac{3\hat{\alpha}}{\hat{\omega}} \dot{\hat{e}}'_1 - L_{21}(\hat{e}_1 - z) \\
\dot{\hat{\omega}} &= \dot{\hat{\alpha}} + L_{31}\hat{e}_1(\hat{e}_1 - z) \\
\dot{\hat{\alpha}} &= -L_{41}\dot{\hat{e}}'_1(\hat{e}_1 - z)
\end{align*}
\]

Note: The observer works only within the window when $u_1 \neq u_2 = u_3$
otherwise, $L_{11} = L_{21} = L_{31} = L_{41} = 0$
Figure 12: The estimated frequency obtained from the NAO taking into account that $z \rightarrow e_1$ for $\delta = 0.1ms$, $L_{11} = 4000$, $L_{21} = 4 \times 10^6$, $L_{31} = 6000$ and $L_{41} = 150$
Observers with Battery Current Measurements

Sliding mode observer (step 1)

\[ i_{\text{battery}} = i_{\text{link}} - i_{\text{load}} \]
\[ i_{\text{load}} \approx \text{const.} \]
\[ u_1 i_{\text{link}} = i_1 \]

\[
\frac{d}{dt} \hat{i}_1 = -\frac{R_w}{L} \hat{i}_1 - \frac{v_o}{6L} (2u_1 - \sum_{j \neq 1} u_j) + \frac{1}{L} M_1 \text{sign}(u_1 i_{\text{battery}} - \hat{i}_1)
\]

\[
\dot{s}_1 = i_1 - u_1 i_{\text{load}} - \hat{i}_1 = -\frac{1}{L} \left[ R_w s - e_1 + M_1 \text{sign}(s_1) + R_w u_1 i_{\text{load}} + u_1 L i_{\text{load}} \right]
\]

\[
(M_1 \text{sign}(i_1))_{eq} \Rightarrow z = e_1 - u_1 q; \quad q = R_w j_{\text{load}} = \text{const.}
\]
Nonlinear Asymptotic Observer with Battery Current Measurements

\[ z = e_1 - u_1 q \]

\[
\begin{aligned}
\dot{\hat{e}}_1 &= \hat{\dot{e}}_1^\prime - L_{11} \bar{e}_1 \\
\hat{\dot{e}}_1^\prime &= -(\bar{\omega}^2 + \frac{3\hat{\alpha}^2}{\bar{\omega}^2}) \hat{e}_1 - \frac{3\hat{\alpha}}{\bar{\omega}} \hat{\dot{e}}_1^\prime - L_{21} \bar{e}_1 \\
\bar{\omega} &= \hat{\alpha} + L_{31} \hat{e}_1 \bar{e}_1 \\
\hat{\alpha} &= -L_{41} \hat{\dot{e}}_1^\prime \bar{e}_1 \\
\hat{q} &= L_{51} u_1 \bar{e}_1; \quad \bar{e}_1 = \hat{e}_1 - z - u_1 \hat{q}
\end{aligned}
\]

under the assumption

\[ \dot{q} = 0, \quad q = R_w i_{load} \]
Figure 46: Schematic diagram of experimental apparatus
Both Estimated Back EMFs Based on Measured Phase and Link Currents

Figure 68: The comparison of both estimated back EMFs based on the phase and link current measurements