M7 : Control and stabilisation of PDE systems:
Theory and application to physical systems and flexible structures

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Course description:

The general context of this course is the study and the control of infinite dimensional systems. This class of systems appear in various applications when a model with an infinite (or a large) number of input/state/output variables should be considered. We focus on the control and the stabilisation properties of two kinds of partial differential equations (PDE). For each class of PDE, one general application is highlighted: the control of physical networks and the control of flexible structures. Let us shortly motivate the study of each applications.

The operation of many physical networks having an engineering relevance may be represented by hyperbolic systems of balance laws in one space dimension. Among the potential applications we have in mind, we mention for instance hydraulic networks (for irrigation or navigation) or road traffic networks. In each of these applications, the network is represented by a directed graph with edges and nodes. Along the edges, the dynamics of the concerned physical quantities are modeled by hyperbolic PDEs under the form of so-called 2x2 systems of balance laws. The nodes of the graph represent the physical junctions between some of the edges of the network. The mechanisms occurring at these junctions are modeled by algebraic relations that determine the boundary conditions of the PDEs. They generally depend on network physical constraints but, in many instances, they can also be assigned by using appropriate control devices (like hydraulic gates in open channels or traffic lights in road networks).

The control and the description of flexible structures appear in various applications like in aeronautic systems, or spatial devices. For this class of physical systems, it is often possible to equip the dynamical structures with smart materials (which are devices which convert electric energy into forces e.g.). The distribution of such smart actuators and sensors may be diffuse and their number may be so large that it may be necessary to consider a PDE model to describe the dynamics and the input/output properties. We consider the controllability and the stabilisability properties of such models. Two kinds of applications are considered. Firstly the active reduction of vibrations of flexible structures, and secondly the reshape problem of flexible structures.

In the first part of this course, we give some physical motivations of the study of hyperbolic systems. We also define the associate control problem, and we introduce one of the main ingredients of this PDE which is the existence of the Riemann coordinates. This allows us to solve the stabilisation problem. Some applications will be given for navigable waterways and for road traffic networks.
In the second part of this course, we consider the control and the stabilisation of flexible structures. Different techniques will be considered, firstly the Hilbert Uniqueness Method (HUM) and secondly the multiplier method. These techniques are illustrated on the study of a beam equipped with a piezoelectric actuator and on flexible plates. More precisely, as for controlled plates, we consider the case of a bimorph mirror in adaptative optics and the case of smart plate in interaction with a fluid.

The course is suitable for engineering and mathematics students who are familiar with basic linear control system theory.

Outline of the course (tentative):

Part I: Physical networks

- Scalar conservation laws
- Systems of two conservations laws
- Characteristic form and Riemann invariants
- Statement of the boundary feedback control problem
- Simulation results
- Lyapunov stability analysis
- Control with integral actions
- Generalisation to networks of balance laws

• Application to networks of navigable waterways
• Networks of scalar laws. Application to road traffic networks
• Boundary feedforward control
• Design of state observers
• Robustness issues
• Illustration by numerical simulations and on an experimental channel;
• Control and stabilisation of a moving tank.

Part II: Flexible structures

• The beam equation
• The piezoelectric phenomenon
• Controllability of the smart beam equation
• A robust controller for the active reduction of vibrations
• Stabilisability of the smart beam equation
• Speeds of convergence towards the equilibrium
• The plate equation
• The adaptive optics problem
• A robust controller for the bimorph mirrors
• Numerical simulations of an adaptive optics system
• A fluid-structure system with smart materials
• Numerical simulations and experiments with a robust controller.

Georges Bastin is Professor in the Department of Mathematical Engineering at Universit\'e Catholique de Louvain and former Associate Professor at Ecole des Mines de Paris.

His main research interests are in nonlinear control of compartmental systems and boundary control of hyperbolic systems with applications in biology, robotics, communication networks and environmental systems.

Christophe PRIEUR was born in Essey-les-Nancy, France, in 1974. He graduated in Mathematics from the Ecole Normale Supérieure de Cachan, France in 2000. He received the Ph.D degree in 2001 in Applied Mathematics from the Université Paris-Sud, France. From 2002 he is research associate CNRS at the Laboratoire SATIE, Ecole Normale Supérieure de Cachan, France and from 2004 at the LAAS-CNRS, Toulouse, France.

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