This is a course for graduate students or researchers with a background in linear algebra, convex optimization and some knowledge in linear control systems. The focus is on semidefinite programming (SDP), or linear matrix inequality (LMI) optimization, and its interplay with semialgebraic geometry, the study of polynomial inequalities.

Outline

The course starts with fundamental mathematical features of linear matrix inequalities:
• Part I.0: general introduction, course outline and material
• Part I.1: historical developments of LMIs, convexity, cones, duality, semidefinite programming

Then we cover latest achievements in semidefinite programming and its interplay with semialgebraic geometry:
• Part I.2: classification of convex semialgebraic sets that can be represented with LMIs and projections of LMIs, lift and project techniques
• Part I.3: primal problem of moments, dual polynomial sum-of-squares decompositions, representations of polynomials positive on semialgebraic sets, Lasserre's hierarchy of LMI relaxations to solve non-convex polynomial optimisation problems

Then we survey polynomial methods in control engineering:
• Part II.1: stability of a polynomial, semialgebraic formulations, geometry of stability conditions
• Part II.2: robust stability of a polynomial
• Part II.3: LMI methods for robust stability and robust stabilization of polynomials, robust fixed-order controller design

Finally, we focus on numerical and software aspects:
• Part III.1: basics of interior-point algorithms, latest achievements in software and solvers for LMIs
• Part III.2: software for polynomial optimization, including polynomial matrix inequalities (PMIs)